SELF-CONSISTENT MOTION OF GAS HEATED BY POINT-LIKE ISOTROPIC SOURCE OF MONOCHROMATIC RADIATION

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Spherically symmetric motion is considered of an ideal gas with the initial distributions

$$v(r,\ 0) = 0;\ \rho(r,\ 0) = \rho_0;\ \varepsilon(r,\ 0) = A/r^2,$$

where v is mass velocity; ρ is density; and ε is specific energy. For $t > 0$ one has

$$v(0,\ t) = 0;\ v(r \to \infty,\ t) = 0.$$

These conditions arise, for example, in the case of instantaneous heating of gas by a point-like isotropic monochromatic source of radiation if the initial heating can be represented by

$$\varepsilon(r,\ 0) = (E_0/4\pi r^2 L\rho_0)e^{-r/L},$$

where L is the path length; E_0 is the total radiation energy. For $r \ll L$ one has

$$\varepsilon(r,\ 0) = A/r^2,\ A = E_0/4\pi L\rho_0.$$

In the above formulation the problem contains only two dimensional parameters, A and ρ_0, from which it is impossible to work out the parameter of the length dimension. Therefore, the motion of the gas for $t > 0$ is self-consistent (self-consistency of the first kind).

Of course, the flows are self-consistent for any power formula $\varepsilon(r,\ 0) = Ar^{-n}$. However, the values $n \geq 3$ are physically not feasible, since they correspond to the case of infinite energy in a sphere of finite radius.

If the notation of [1] are adopted, then the self-consistent variable can be written as

$$\lambda = r/A^{1/4}t^{1/2}. \tag{1}$$

Following [1], the dimensionless variables V, R, and P are introduced by means of

$$v = (r/t)V;\ \rho = \rho_0 R;\ p = \rho_0(r^2/t^2)P, \tag{2}$$

where p is pressure.

By substituting (1) and (2) in the equations of gasdynamics, one obtains

$$\begin{aligned} &\lambda[(1/2 - V)V' - P'/R] = V^2 - V + 2P/R;\\ &\lambda[-V' + (1/2 - V)R'/R] = 3V;\\ &\lambda(1/2 - V)[P'/P - \gamma R'/R] = 2V - 2 \end{aligned} \tag{3}$$

for the spherical-symmetry case having introduced the variable Z by means of

$$Z = \gamma P/R = \gamma(p/\rho)t^2/r^2, \tag{4}$$

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TABLE 1

γ	λ^*	V_1^*	V_2^*	P_1^*	P_2^*	R_1^*	R_2^*
1,1	1,284	0,0945	0,376	0,0437	0,1664	1,076	3,512
1,2	1,536	0,0921	0,357	0,0427	1,1585	1,074	3,056
1,4	1,847	0,0886	0,323	0,0412	0,1442	1,0711	2,483
1,6667	2,133	0,0828	0,287	0,0387	0,1294	1,0672	2,086

where γ is the adiabatic exponent; one obtains from (3) the differential equations for $Z(V)$,

$$\frac{dZ}{dV}=\frac{Z\left\{[2(V-1)+3(\gamma-1)V]\left(V-\frac{1}{2}\right)^2-(\gamma-1)V(V-1)\left(V-\frac{1}{2}\right)\right\}-\left[2(V-1)+\frac{1}{\gamma}(\gamma-1)\right]Z^2}{\left(V-\frac{1}{2}\right)\left[V(V-1)\left(V-\frac{1}{2}\right)+\left(\frac{1}{\gamma}-3V\right)Z\right]}. \quad (5)$$

Bearing in mind the character of the initial conditions, it is assumed that the flow contains a shock wave. To the shock front r^*, which is propagated over the heated gas in motion, there is associated a specific value of λ^*, which depends on γ, namely,

$$r^*(t)=\lambda^* A^{1/4}\sqrt{t}. \quad (6)$$

The parameters at the front of the shock wave are denoted below by an asterisk.

One obtains an asymptotic solution of the system of equations (3) for $\lambda\to\infty$ by assuming that $\rho\to\rho_0$, $v\to 0$ (that is, $R\to 1$, $V\to 0$)

$$V=2(\gamma-1)/\lambda^4+(\gamma-1)^2(6\gamma+22)/3\lambda^8;\ P=(\gamma-1)/\lambda^4+(\gamma-1)^2(\gamma+2)/\lambda^8;$$
$$R=1+(\gamma-1)/\lambda^4+5(\gamma-1)^2(3\gamma+14)/6\lambda^8,$$

where the expansion terms $\sim\lambda^{-12}$ have been neglected. For $\lambda\to 0$

$$V=1/3\gamma+[(3\gamma-1)(3\gamma-2)^2/54\gamma^4(15\gamma-4)]C_2/C_1\cdot\lambda^{(2+6\gamma)/(3\gamma-2)};$$
$$P=C_1/\lambda^2+[(3\gamma-1)(3\gamma-2)/9\gamma^2(2+6\gamma)]C_2\lambda^{6/(3\gamma-2)};$$
$$R=C_2\lambda^{\frac{6}{3\gamma-2}}+\frac{(15\gamma+2)(3\gamma-1)(3\gamma-2)}{(15\gamma-4)9\gamma^3(2+6\gamma)}\frac{C_2^2}{C_1}\lambda^{\frac{8+6\gamma}{3\gamma-2}},$$

where C_1 and C_2 are constants which can only be determined by solving the problem completely. However, one can find the value of $Z(V)$, which does not contain C_1 and C_2,

$$Z=\frac{(3\gamma-1)(3\gamma-2)^2}{54\gamma^3(15\gamma-4)\left(V-\frac{1}{3\gamma}\right)}\left[\frac{(3\gamma-2)(2+6\gamma)+6\gamma^2(15\gamma-4)\left(V-\frac{1}{3\gamma}\right)}{(3\gamma-2)(2+6\gamma)+6\gamma(15\gamma+2)\left(V-\frac{1}{3\gamma}\right)}\right]. \quad (7)$$

The flow parameters downstream behind the shock wave and upstream in front of it are related by the relations on the nonremovable discontinuity,

$$V_2^*-\frac{1}{2}=\left(V_1^*-\frac{1}{2}\right)\left[1+\frac{2}{\gamma+1}\frac{Z_1^*-\left(V_1^*-\frac{1}{2}\right)^2}{\left(V_1^*-\frac{1}{2}\right)^2}\right];$$
$$Z_2^*=\left(\frac{\gamma-1}{\gamma+1}\right)^2\frac{1}{\left(V_1^*-\frac{1}{2}\right)^2}\left[\left(V_2^*-\frac{1}{2}\right)^2+\frac{2Z_1^*}{\gamma-1}\right]\left[\frac{2\gamma}{\gamma-1}\left(V_1^*-\frac{1}{2}\right)^2-Z_1^*\right]; \quad (8)$$
$$R_2^*\left(V_2^*-\frac{1}{2}\right)=R_1^*\left(V_1^*-\frac{1}{2}\right),$$

where the subscript 1 corresponds to the upstream and 2 to the downstream parameters.

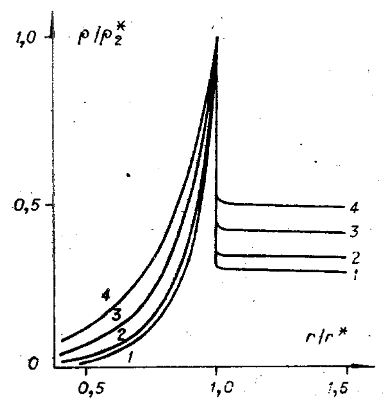

Fig. 1

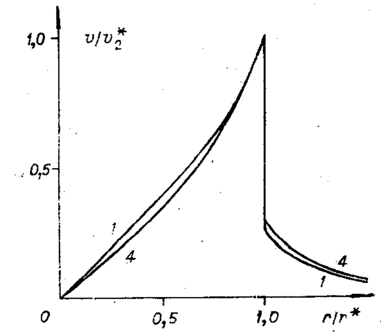

Fig. 2

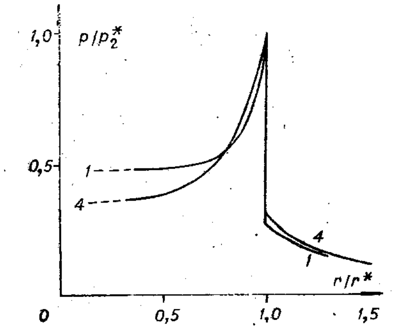

Fig. 3

The functions $R(\lambda)$, $V(\lambda)$, $P(\lambda)$ for any values of λ can only be determined by numerical integration of the ordinary differential equations (3). For some $\lambda_1 \gg 1$ the values of $R(\lambda_1)$, $V(\lambda_1)$, $P(\lambda_1)$ are computed by using the asymptotic formulas (6). Using these values as initial conditions one now integrates numerically the system of equations (3) in the domain $\lambda < \lambda_1$, and one obtains the functions $R_1(\lambda)$, $V_1(\lambda)$, $P_1(\lambda)$; then from the formula (4) the value $Z_1(\lambda)$ is obtained. Substituting $Z_1(\lambda)$ and $V_1(\lambda)$ in the relations (8), one obtains the function $Z_2^+(V_2)$ and for some value $V = V_2^0$ the value of $Z_2(V_2^0)$ is computed by means of the formula (7). [The relation (7) holds for $\lambda \ll 1$, in other words, for $V_2^0 - 1/3\gamma \ll 1$.] Using $Z_2(V_2^0)$ as the initial value for the differential equation (5), one integrates the latter in the domain $V_2 > V_2^0$, and one obtains $Z_2^-(V_2)$. From the relation

$$Z_2^+(V_2(\lambda)) = Z_2^-(V_2)$$

one can determine V_2^* and Z_2^*.

Employing the numerical functions $R_1(\lambda)$, $V_1(\lambda)$, $P_1(\lambda)$ and the conditions (8) at the front of the shock wave one finds the values of λ^*, R_1^*, V_1^*, P_1^*, R_2^* and P_2^*. The integration of (3) for $\lambda < \lambda^*$ with the initial conditions $R_2(\lambda^*) = R_2^*$, $P_2(\lambda^*) = P_2^*$, $V_2(\lambda^*) = V_2^*$ yields the values $R_2(\lambda)$, $V_2(\lambda)$, $P_2(\lambda)$ downstream from the shock wave.

The values λ_1 and V_2^0 have been chosen on the basis of the requirements of the relative error in the computation of the parameters in front of the shock wave as $\sim 10^{-4}$–10^{-3}. In the calculations the values $\lambda_1 = 4$, $V_2^0 = 1/3\gamma + 0.0025$.

In the Table 1 the computation results are given of the parameters at the front of the shock wave for four different values of γ. The profiles of density, velocity, and pressure are shown in Figs. 1-3 (curve 1 corresponds to $\gamma = 1.1$; 2, to $\gamma = 1.2$; 3, to $\gamma = 1.4$; and 4, to $\gamma = 1.6667$).

In view of a weak dependence of $p/p_2^*\,(r/r^*)$ and $v/v_2^*\,(r/r^*)$ on γ (in the range $\gamma = 1.1$–$5/3$) these functions are shown in Figs. 2 and 3 only for $\gamma = 1.1$; $5/3$.

The limiting compression is not reached at the shock front propagated over heated gas in motion. For the problem under consideration when $\gamma = 5/3$ the compression $\rho_2^*/\rho_1^* = 1.95$ takes place, whereas the limiting value amounts to 4. The velocity profile downstream from the shock wave is nearly linear. For a finite t and $r \to 0$ one has $\rho(r) \to 0$, $p(r) \to$ const.

The region of applicability of the obtained solution is bounded by the conditions

$$r \ll L \text{ and } r \ll \sqrt{E_0/4\pi L \rho_0 \varepsilon_0}. \tag{9}$$

If the first condition is satisfied, one can represent the initial heating in the form of specific energy being a power of the radius; if the second condition is fulfilled, the intrinsic energy ε_0 of the cold gas can be ignored as compared with the heating energy of the gas by radiation. The self-consistent solution is valid as long as the front radius of the shock wave satisfies the relations (8).

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EXPERIMENTAL STUDY OF STRONG SHOCK PROPAGATION IN A CHANNEL

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The widespread use of shock tubes in laboratory practice is well known. However, despite existing information [1] about shock-wave velocities of ~ 100 km/sec, experimental data on the size of the shock-heated region behind the shock front are confined to the Mach numbers M = 10 [2]. Theoretical data do not go beyond the limit of this range except for air where calculations were performed up to M ≈ 20 [3, 4]. Behind strong shocks, the effects resulting from viscosity, thermal conductivity, and radiation of the medium should lead to serious deviation of the actual flow from the idealized pattern for uniform motion of a piston in a channel filled with a nonviscous, thermally nonconducting, and nonradiating medium. It is therefore practical to make an experimental study of the behavior of density and of the size of the shock-heated region behind a shock front propagating down the channel of a shock tube that is capable of producing velocities up to 8 km/sec.

The shock tube used in the experiments was similar to the one described in [5], but in these experiments the internal cross section of the channel in the low-pressure chamber was a 27 × 27 mm^2. An additional section of greater cross section was installed beyond the viewing ports so that reflection of the shock wave from the end did not disturb the flow patterns. Argon was used as the test gas.

The study was based on the interferometric method for determination of density using the Rozhdestvenskii system [6] in which a Mach–Zender interferometer was crossed with a spectrograph. This system was supplemented with a time sweep similar to that used in [7]. The experimental arrangement is shown in Fig. 1. In the figure, I-IV are interferometer mirrors; L_1 and L_2 are lenses; 6 is an ISP-51 spectrograph; 7 and 8 are a driven photodetector and its supply unit; 5 and 4 are the light source and its supply unit.

A pulsed discharge in a capillary was used as the probing light source 5; the energy distribution in its spectrum corresponded to the distribution in a black-body spectrum at a temperature of approximately 40,000°K. A description of this source is given in [8].

The system for velocity measurement is shown schematically in Fig. 1. Here D_1, D_2, D_3, D_4, and D_5 are ionization detectors with the distances D_1D_2, D_2D_3, D_3D_4, and D_4D_5 being respectively, 100, 300, 80, and 240 mm; 1 is a unit which produces an electrical impulse after shorting of the interelectrode gaps of the ionization detectors by plasma; 2 is an S1-17 oscilloscope; 3 is a G-4-18A sine-wave generator; 4 is a synchronization unit.

The driven photodetector consisted of an objective, a 12-sided prism with mirror faces, and a film cassette (Isopankhrom-13 film was ususally used). The rate of prism rotation during an experiment was 375 rps, which made it possible to obtain a time resolution of about 0.3 μsec. Since high-order interference was used, nearly vertical bands were produced on the film. When one of the arms of the interferometer was crossed by a wave or by the contact region, a shift in the interference pattern occurred. Detection of these points in time on the film made it possible to determine the distance between the shock wave and the contact region.

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